# Quarkonium Production at $ep$ and $e^+e^-$ Colliders

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- Highlights from Last Week's Episode
- $\gamma\gamma \to J/\psi + X$ at LEP
- Inelastic $J/\psi$ Photoproduction Cross Section at HERA
- Polarization in Inelastic $J/\psi$ Photoproduction at HERA
- $J/\psi$ Production in DIS at HERA
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- Summary

# Highlights from Last Week's Episode

## NRQCD Factorization Formula

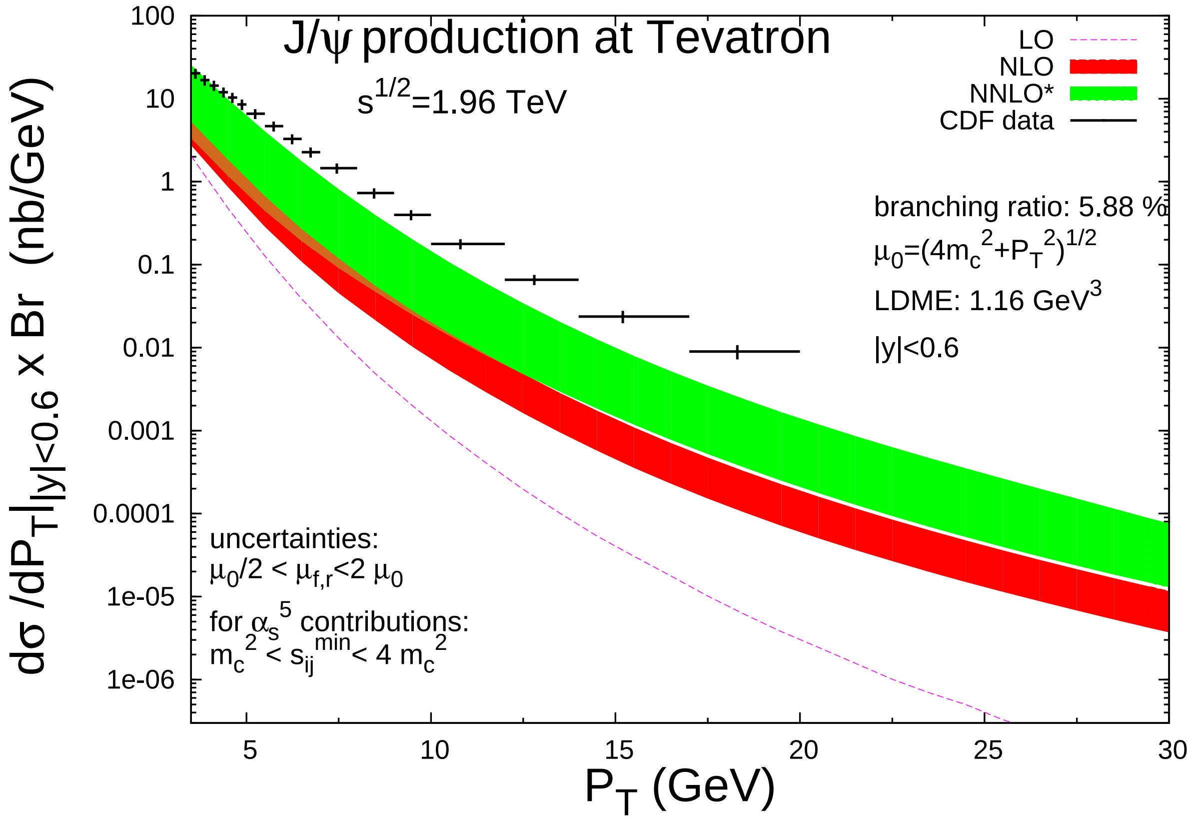
- Conjecture (GTB, Braaten, Lepage (1995)):
  The inclusive cross section for producing a quarkonium at large momentum transfer ($p_T$) can be written as a sum of "short-distance" coefficients times NRQCD matrix elements.

$$\sigma(H) = \sum_n F_n(\Lambda)\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle.$$

- The "short-distance" coefficients $F_n(\Lambda)$ have an expansion in powers of $\alpha_s$.
- The operator matrix elements $\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$ are universal (process independent).
  - Only the color-singlet production and decay matrix elements are simply related.
- The matrix elements have a known scaling with $v$.
- The NRQCD factorization formula is a double expansion in powers of $\alpha_s$ and $v$.
- Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).
  - Inconsistent for $P$-wave production: IR divergent.

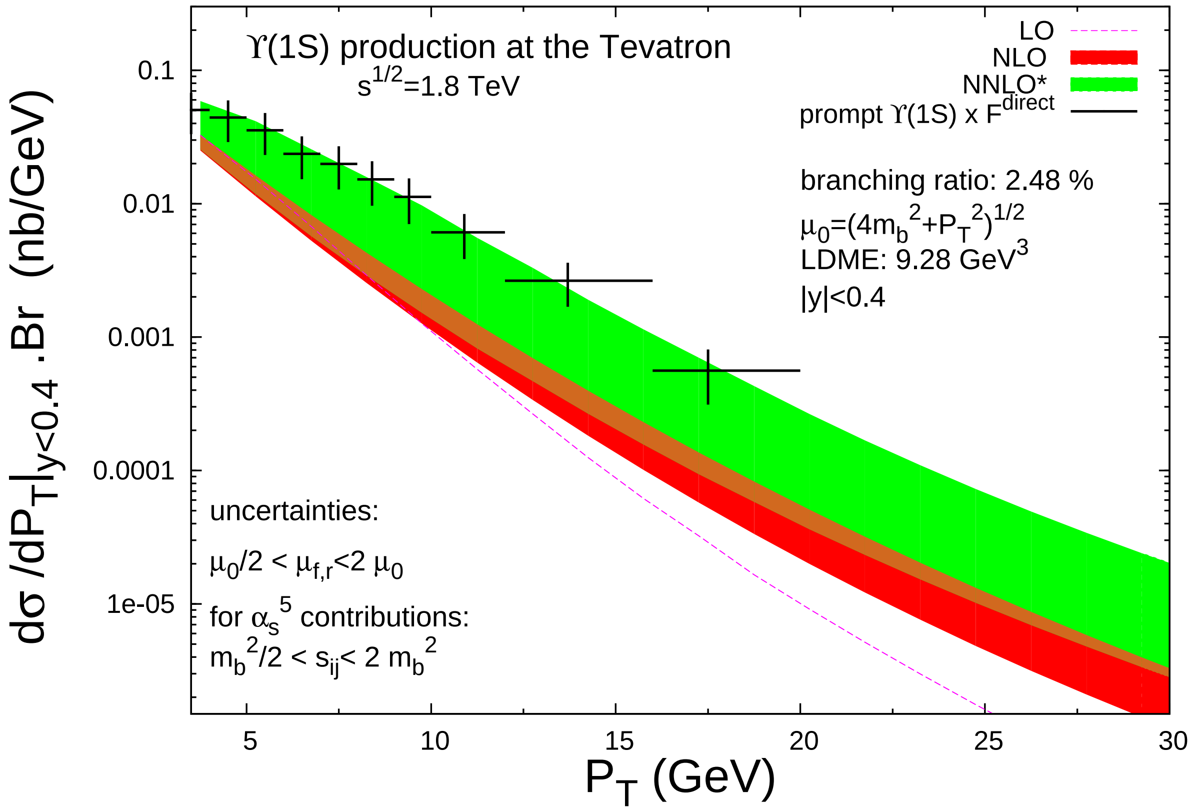
## NLO and NNLO* Contributions to Color-Singlet Production

- Large corrections are caused by slower fall-off with $p_T$ as new channels open up.
- The perturbation expansion might be brought under better control by making use of the fragmentation approach of Kang, Qiu, and Sterman (2010).
- Even if one includes NNLO* corrections to the color-singlet contribution, there is still room for a large color-octet contribution.




- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano.
- The NNLO* calculation is an estimate based on real-emission contributions only.

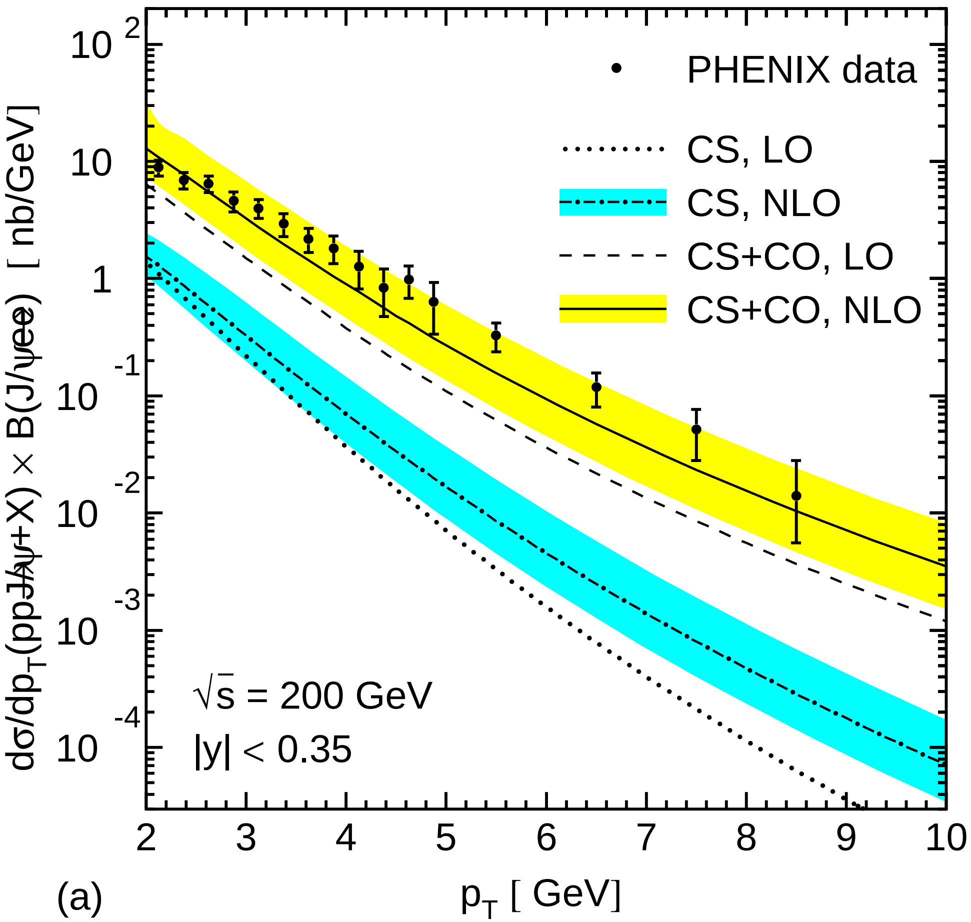
- The NNLO* color-singlet contribution to $\Upsilon$ production could explain the data by itself, but it does not rule out a large, or even dominant, color-octet contribution.



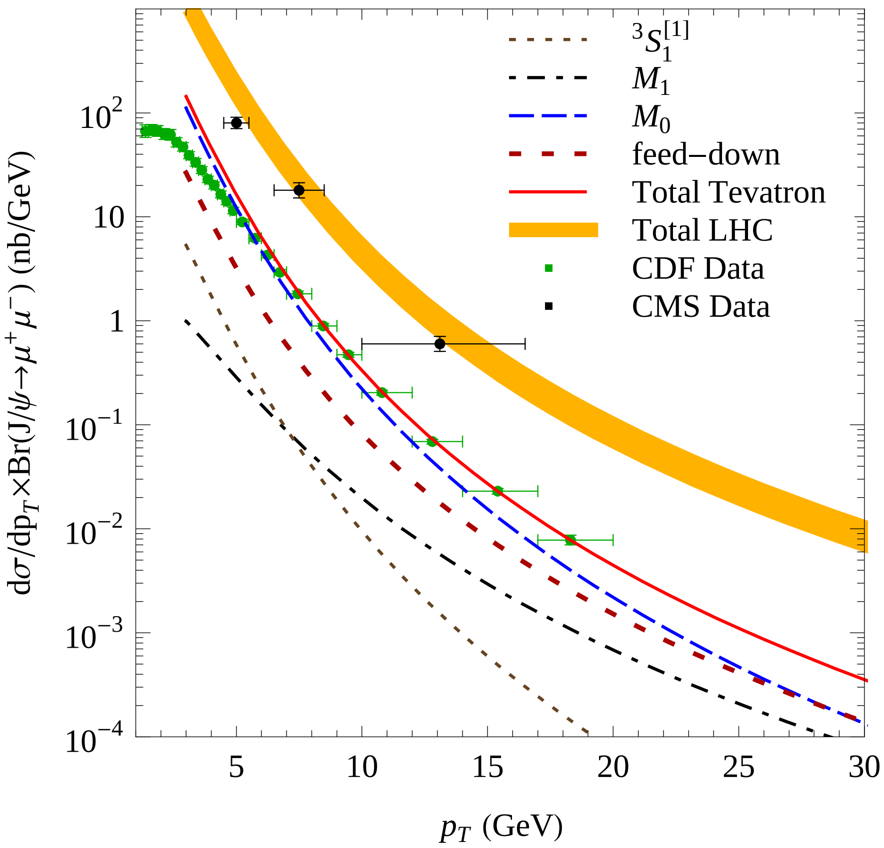

- Plot from Pierre Artoisenet, based on work by Artoisenet, Campbell, Lansberg, Maltoni, Tramontano (2008)
- NLO results confirmed by Gong and Wang (2007).

## NLO Contributions to Color-Octet Production

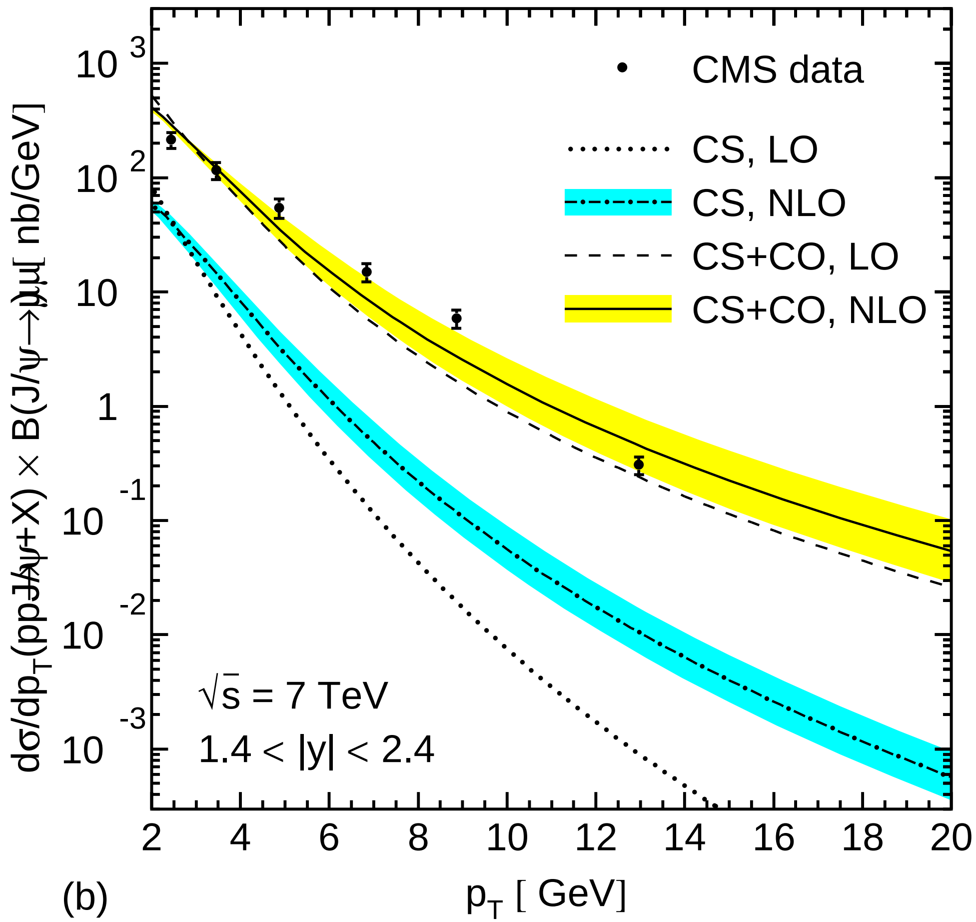
- The first complete NLO calculations of the color-octet contributions through order $v^4$ have been completed recently.
- Corrections to $S$-wave production are small.
  Corrections to $P$-wave production are large.
- Color-octet matrix elements that were obtained from fits to the Tevatron data lead to predictions for $J/\psi$ production at RHIC and the LHC that are in good agreement with the data.




- NLO NRQCD calculation of Kniehl and Butenschön (2010).
- Feeddown ($\approx 36\%$) is not included in the theoretical prediction.
- The NLO color-singlet contribution is well below the PHENIX data.




Ma, Wang, and Chao (2010)
$|y| < 2.4$

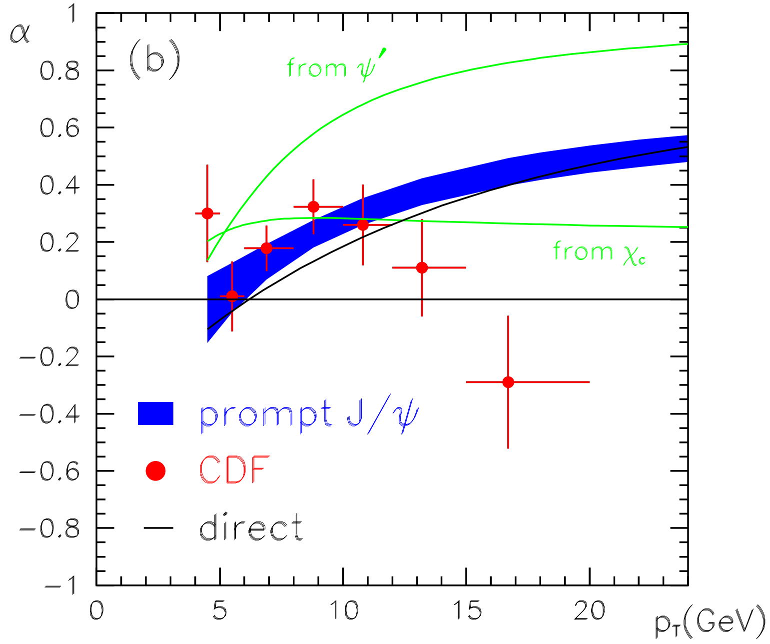



Butenschön and Kniehl (2010)
$1.4 < |y| < 2.4$

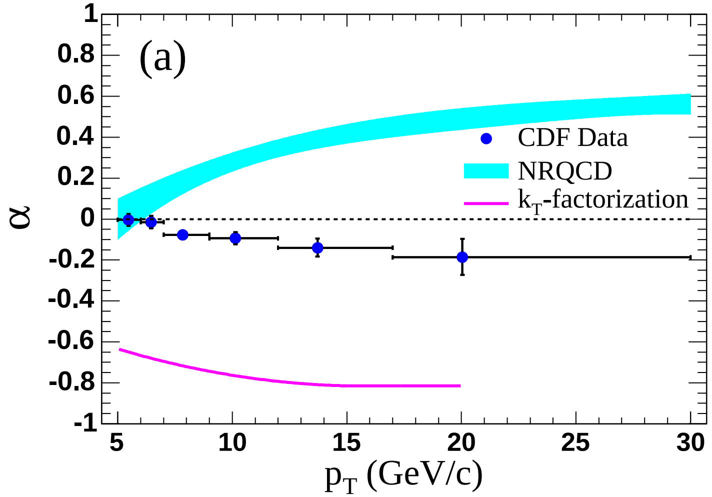
# Polarization

- LO NRQCD predictions for polarization predict large transverse polarization at large $p_T$.
- This prediction has not been borne out by the data.

## Run I $J/\psi$ polarization:

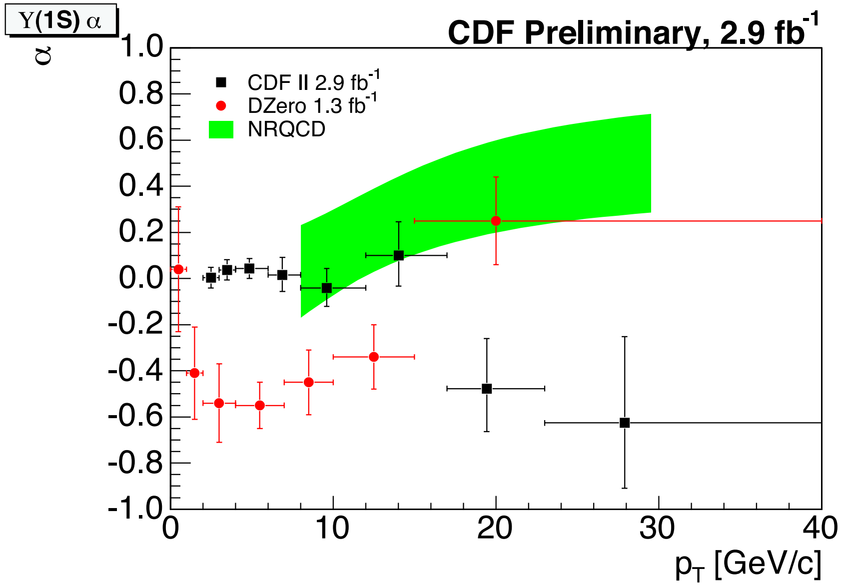



## Run II $J/\psi$ polarization:

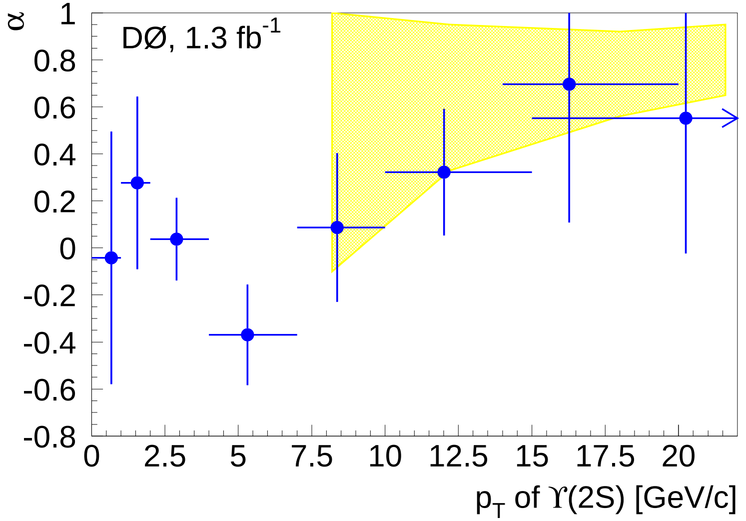



- $d\sigma/d(\cos\theta) \propto 1 + \alpha\cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.
- NRQCD prediction from Braaten, Kniehl, Lee (1999).
  - Feeddown from $\chi_c$ states is about 30% of the $J/\psi$ sample and dilutes the polarization.
  - Feeddown from $\psi(2S)$ is about 10% of the $J/\psi$ sample and is largely transversely polarized.
- Run I results are marginally compatible with the NRQCD prediction.
- Run II results are inconsistent with the NRQCD prediction.
- Also inconsistent with the Run I results.
  CDF was unable to track down the source of the Run I-Run II discrepancy.

## $\Upsilon(1S)$ Polarization:




## $\Upsilon(2S)$ Polarization:




- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the LO NRQCD prediction of Braaten and Lee (2000) (green), but in different regions of $p_T$.
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

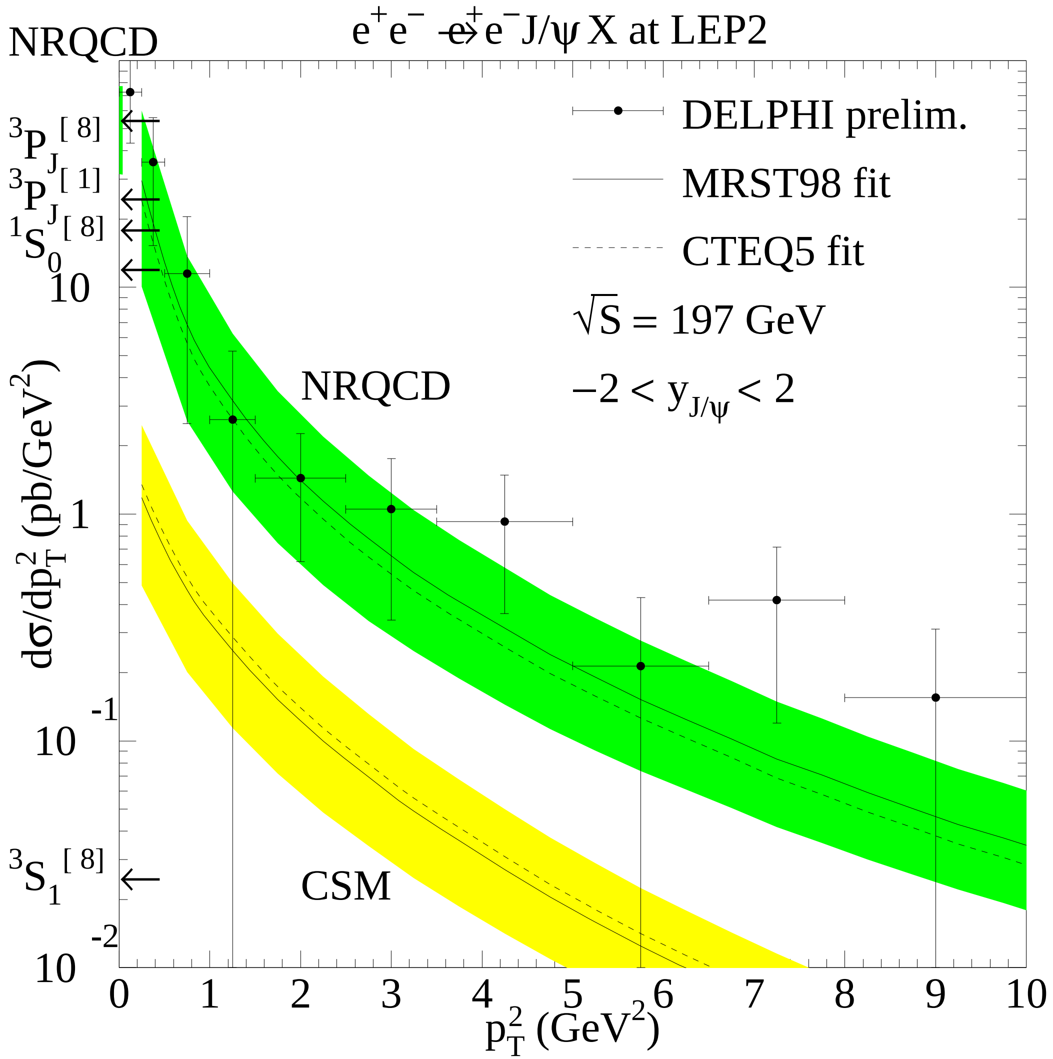
- The experimental results are in considerable disagreement.

## *NLO Corrections to Polarization*

- NLO corrections to $S$-wave color-singlet polarization change the polarization from transverse to longitudinal.
- NLO corrections to $S$-wave color-octet polarization are small.
- Inclusion of (uncalculated) NLO corrections to $P$-wave color-octet processes may have a significant effect on the predictions.
  - Probably not enough to achieve agreement with the data.
    See the talk of Jianxiong Wang.

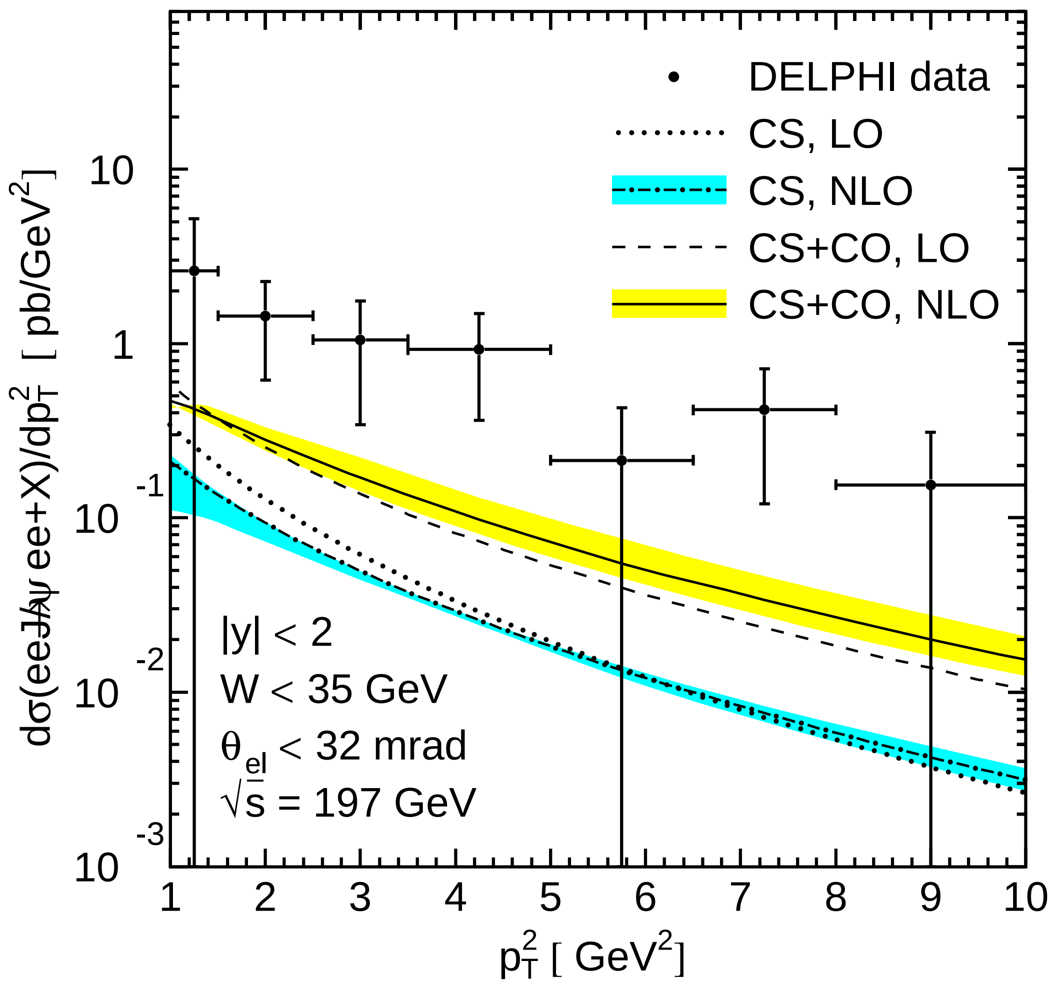
# $\gamma\gamma \to J/\psi + X$ at LEP

- Klasen, Kniehl, Mihaila, Steinhauser (2001): LO NRQCD factorization calculation.




- Comparison with the Delphi (2001) data clearly favors NRQCD over the color-singlet model.
- Theory uses matrix elements fit from CDF data (Braaten-Kniehl-Lee (1999)) and MRST98LO (solid) and CTEQ5L (dashed) PDF's.
- Theoretical uncertainties from
  - Renormalization and factorization scales (varied by a factor 2),
  - NRQCD color-octet matrix elements,
  - Different linear combination of matrix elements than in Tevatron cross sections.

• Butenschön and Kniehl (2011): NLO NRQCD factorization calculation.




• Delphi (2001) data.

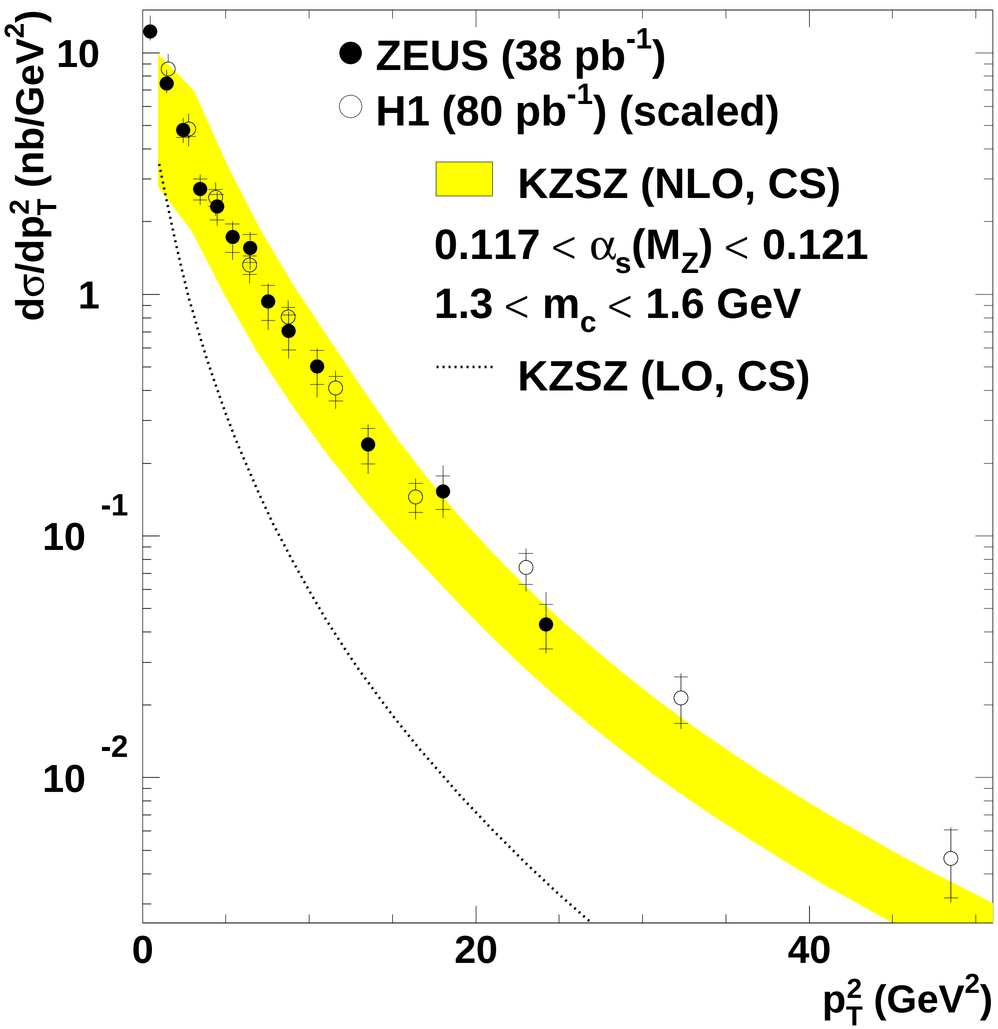
• NRQCD matrix elements determined from a global fit to KEKB, LEP II, RHIC, HERA, Tevatron, and LHC data.

• Uncertainties from factorization, renormalization, and NRQCD scales (varied by a factor 2).

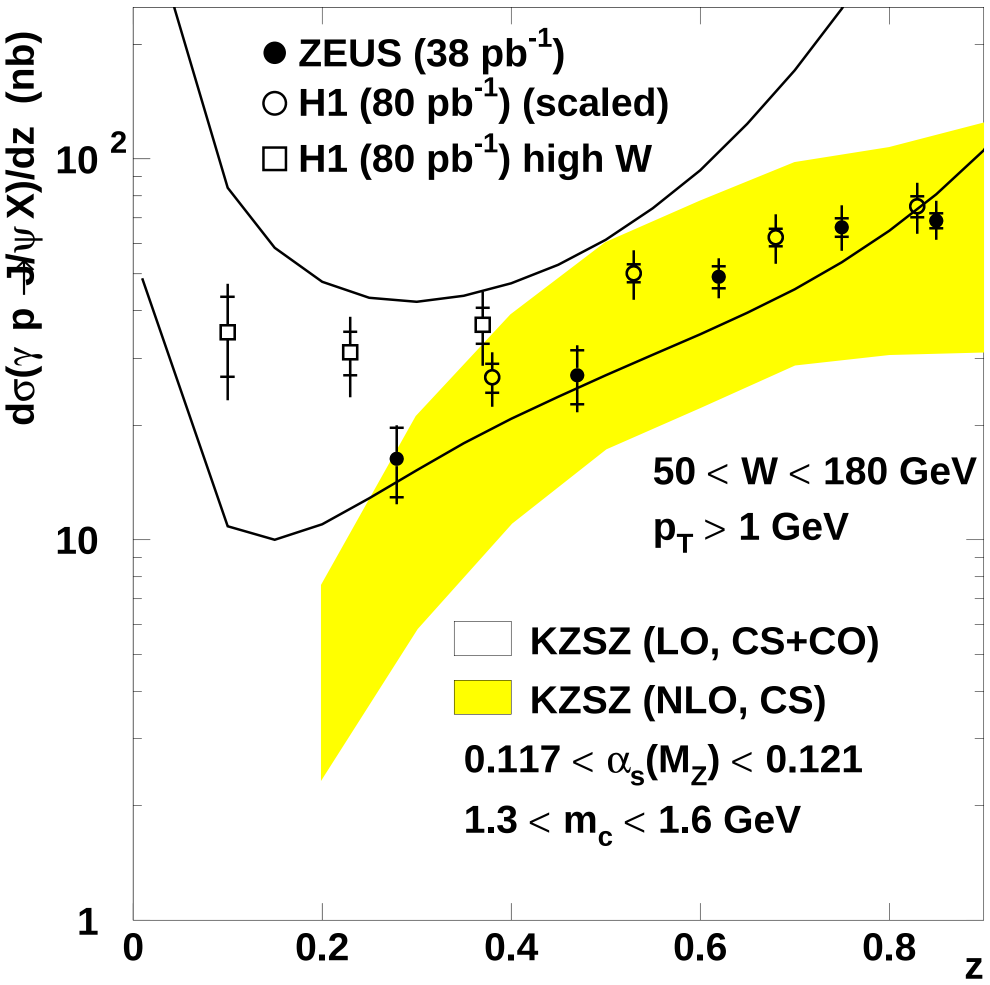
• The NRQCD prediction has shifted down a bit, presumably because the NLO fit to the Tevatron data yields somewhat smaller color-octet matrix elements.

## Inelastic $J/\psi$ Photoproduction Cross Section at HERA

- It had been believed that NLO color-singlet calculations leave little room for a color-octet contribution.



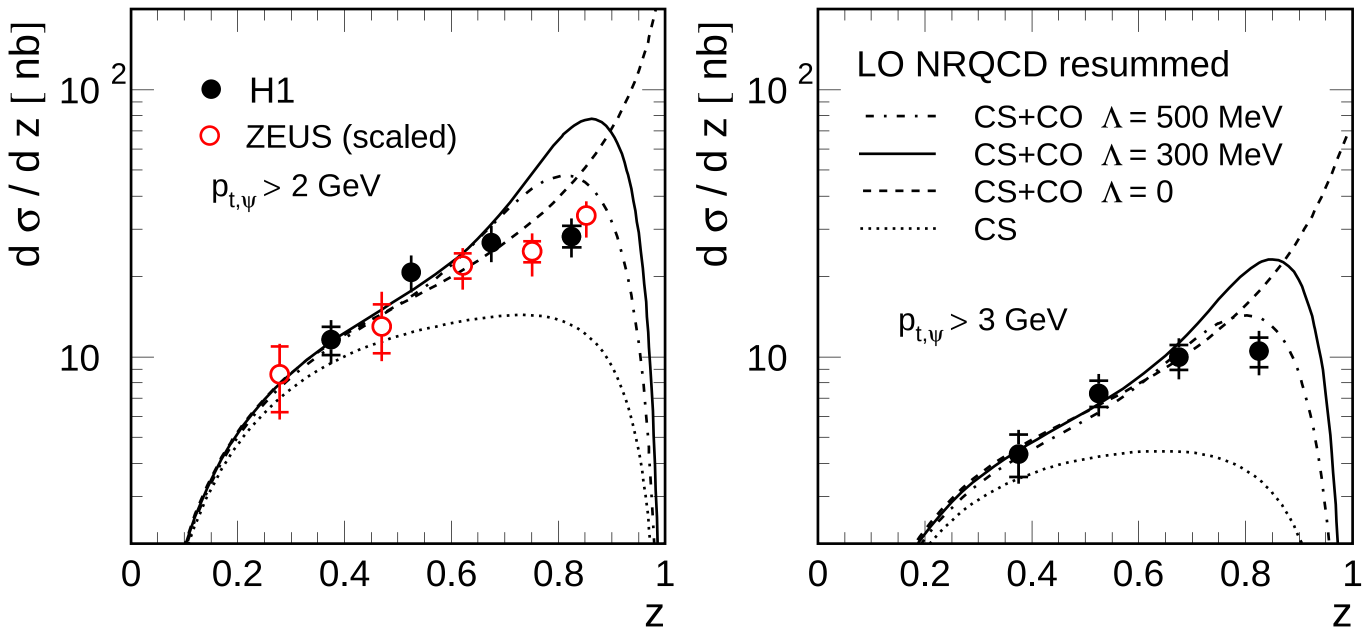

- NLO corrections increase the color-singlet contribution substantially. (Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995))
- NLO corrections include $\gamma + g \to (c\bar{c}) + gg$.
- At large $p_T$, this process goes as $\alpha_s^3 m_c^2/p_T^6$, instead of $\alpha_s^2 m_c^4/p_T^8$.
- Are NNLO corrections also important?




- LO color-singlet plus color-octet calculations by Cacciari, Krämer (1996); Amundson, Fleming, Maksymyk (1996); Ko, Lee, Song (1996); Kniehl, Krämer (1997).
- NLO color-singlet calculations by Krämer, Zunft, Steegborn, Zerwas (1994); Krämer (1995).

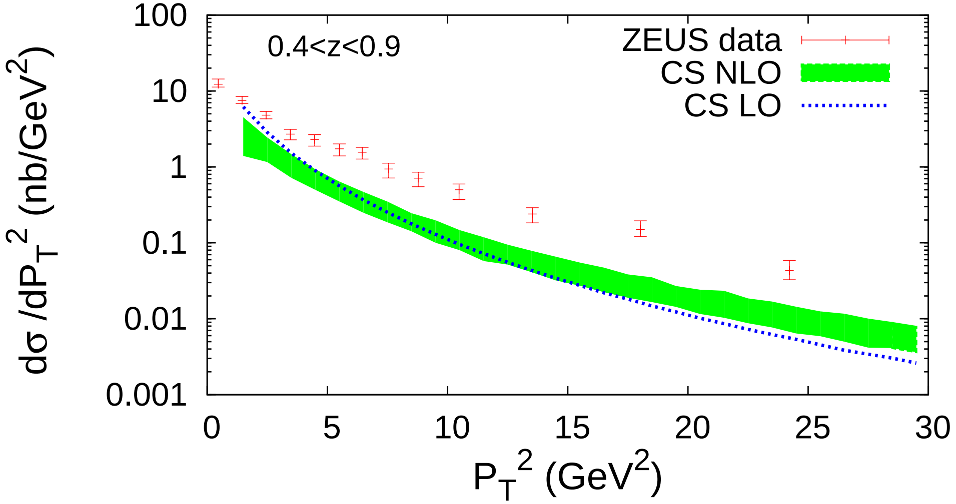
- The $\alpha_s$ expansion breaks down near $z = 1$.
  - Resummation of multiple soft-gluon emission is needed.
    (Beneke, Schuler, Wolf (2000))

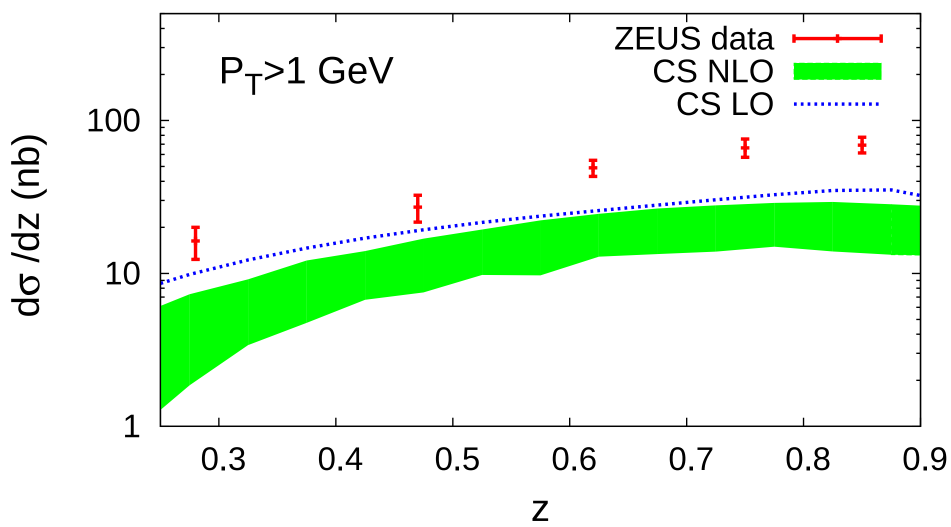
- The $v$ expansion breaks down near $z = 1$.
  - Resummation of the $v$ expansion leads to a nonperturbative shape function.
    (Beneke, Rothstein, Wise (1997))
- Inclusion of a shape function with reasonable choices of parameters leads to an improved fit.



# Recent Theoretical Developments

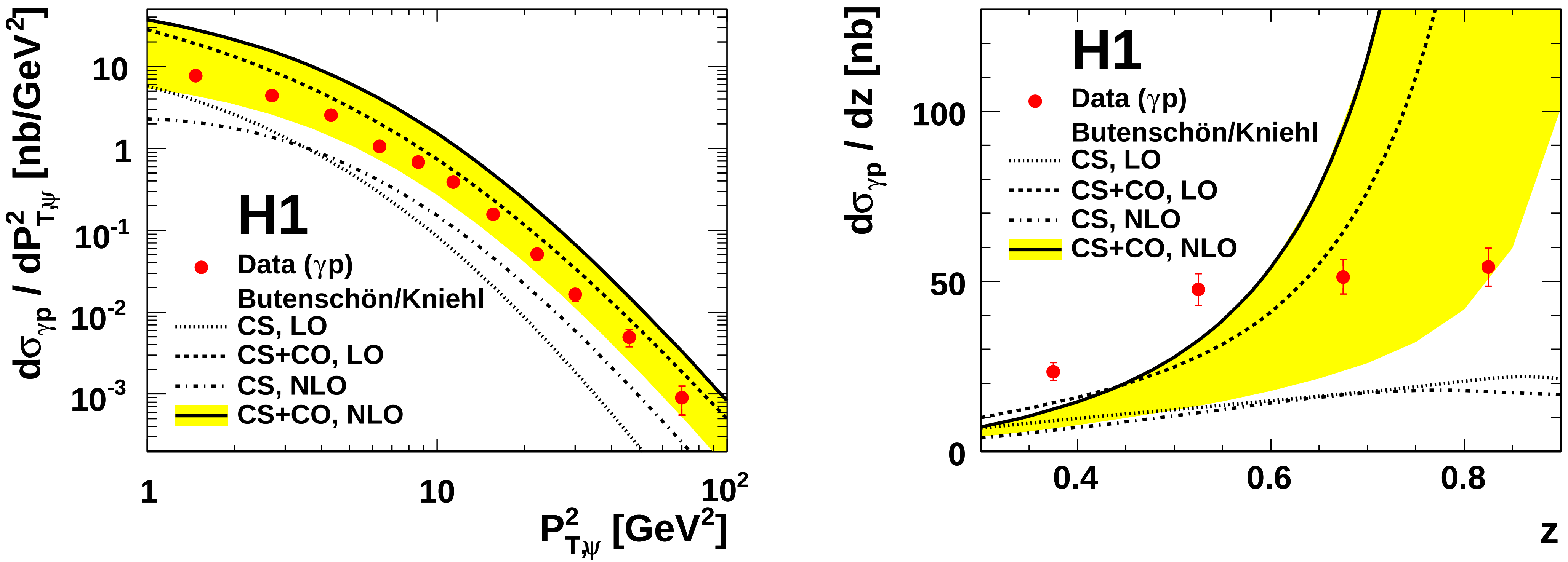
- Artoisenet, Campbell, Maltoni, Tramontano (2009): New calculation of NLO color-singlet contribution
  - Confirms the analytic results of previous calculations.
  - But a more reasonable choice of renormalization/factorization scale ($\sqrt{4m_c^2 + p_T^2}$ instead of $m_c/\sqrt{2}$) yields much smaller numerical results for cross sections.



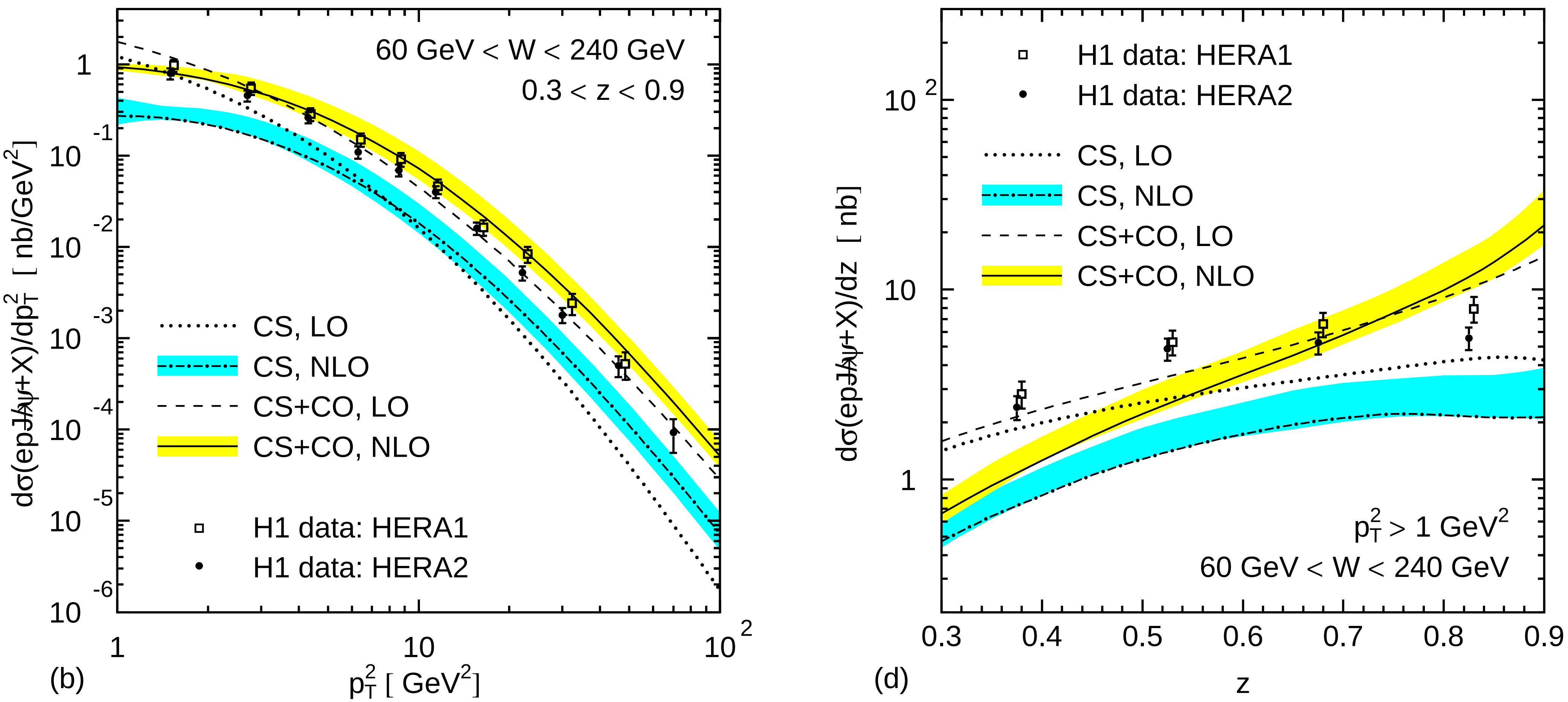

- Leaves room for a color-octet contribution.

- Butenschön and Kniehl (2009): Complete NLO calculation of photoproduction in NRQCD factorization, including both color-octet and color-singlet contributions.
- The color-octet matrix elements are from the fit to the Tevatron data by Kniehl and Kramer (1998), which uses LO plus approximate NLO calculations.

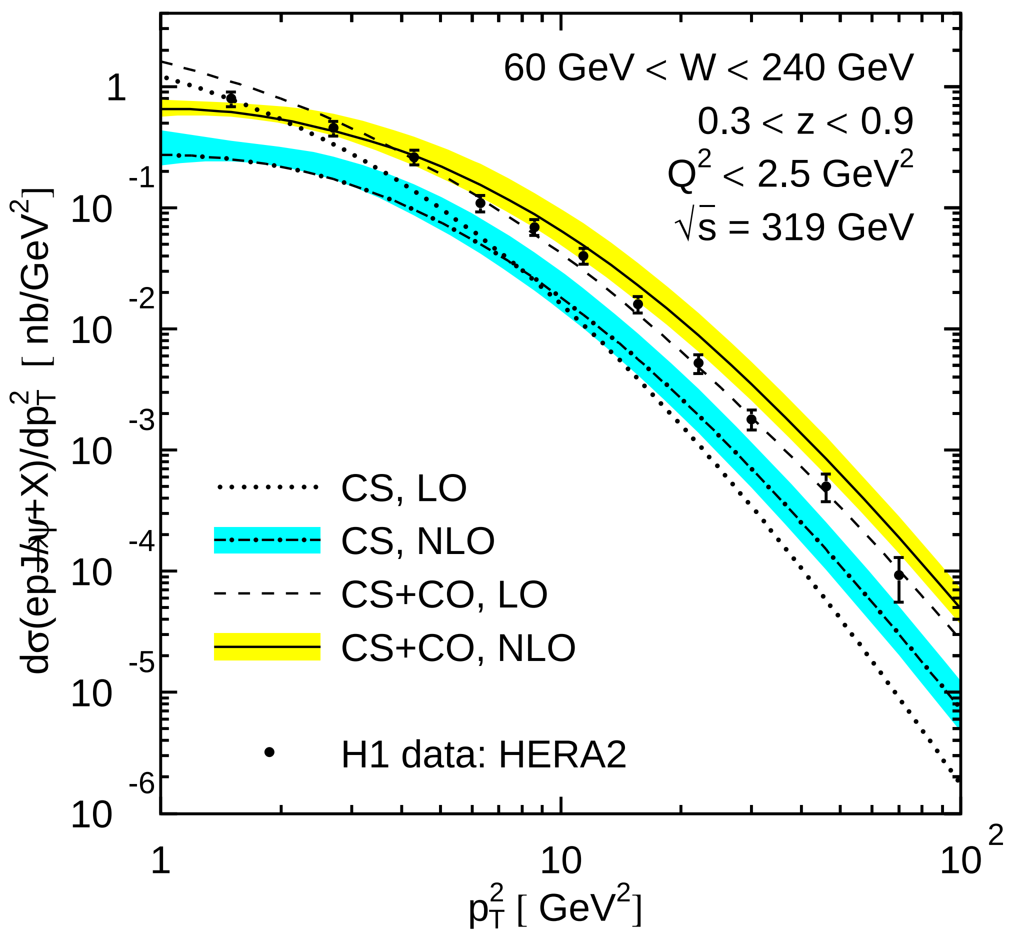


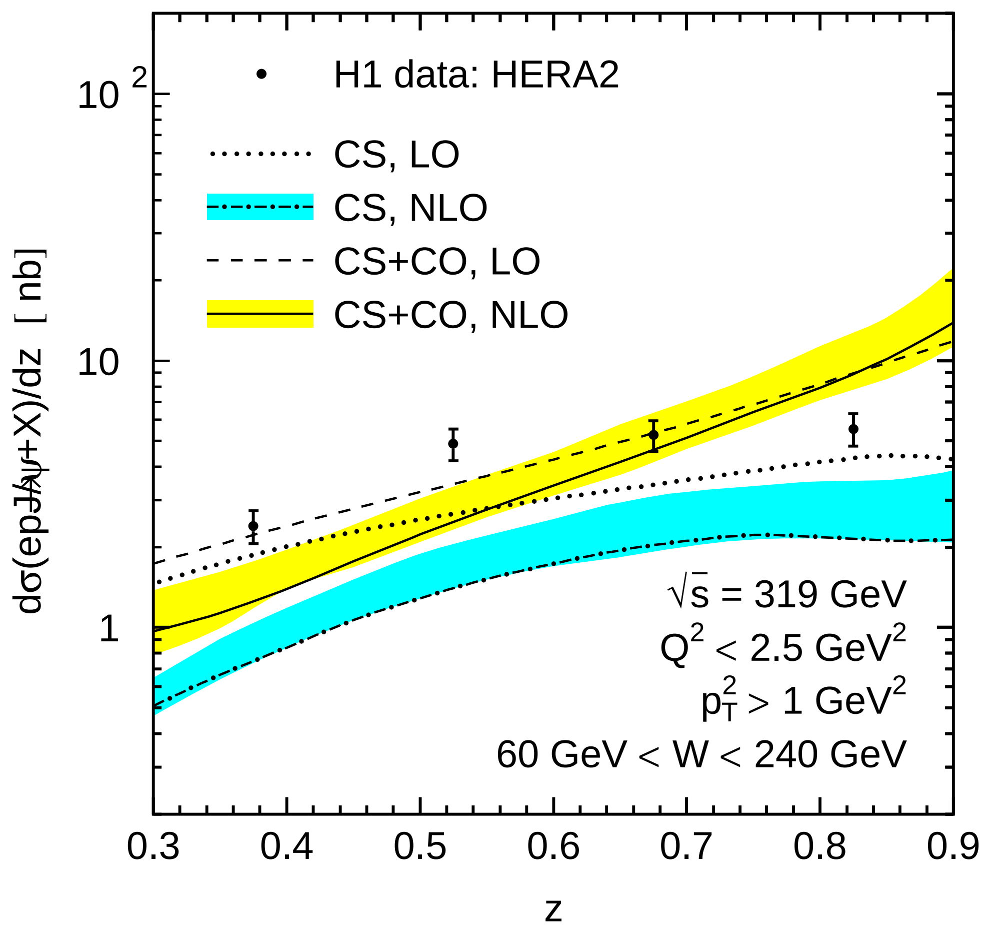

- The comparison of the H1 (2009) data with the Butenschön and Kniehl calculation strongly favors NRQCD factorization over the color-singlet model.
- The discrepancy at low $z$ is due to the omission of resolved contributions.
- The discrepancy at high $z$ would probably be fixed by resummation.

- NRQCD matrix elements from the simultaneous NLO fit to the CDF (2005) and H1 (2009) data yield slightly higher central values and smaller uncertainties.



- Butenschön and Kniehl (2011): NLO calculation, including resolved-photon contributions.



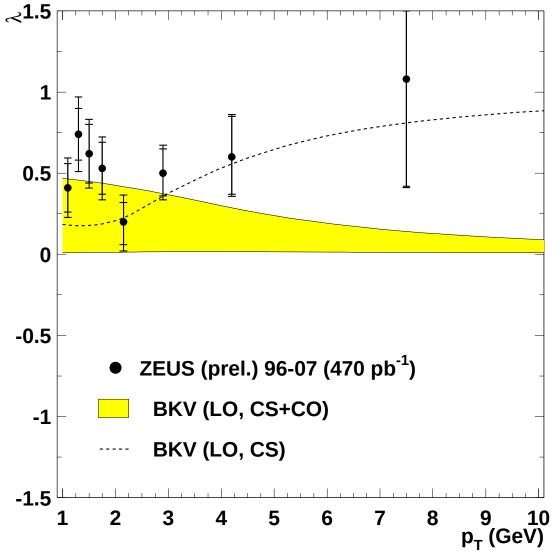



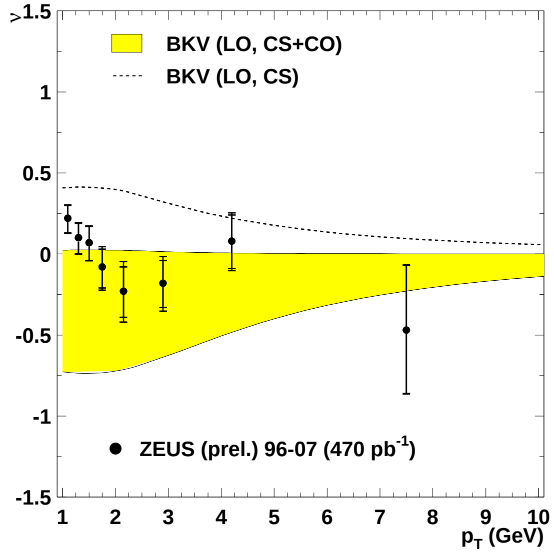

- NRQCD matrix elements from a global fit to KEKB, LEP II, RHIC, HERA, Tevatron, LHC data.
- Uncertainties from factorization, renormalization, and NRQCD scales (varied by a factor 2).
- Inclusion of resolved-photon contributions improves the agreement at small $z$.
- A more negative value of the ${}^3P_0$ color-octet matrix element reduces the rise in cross section near $z = 1$.

# Polarization in Inelastic $J/\psi$ Photoproduction at HERA

$$\frac{d\Gamma(J/\psi\to l^+l^-)}{d\Omega} \propto 1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$

- $\theta$ and $\phi$ are the polar and azimuthal angles of the $l^+$ 3-momentum with respect to the helicity frame.
- The helicity frame is defined in the $J/\psi$ rest frame.
- The $z$ axis is the direction of boost from the lab frame to the $J/\psi$ rest frame.
- $\phi$ is defined with respect to the scattering plane.



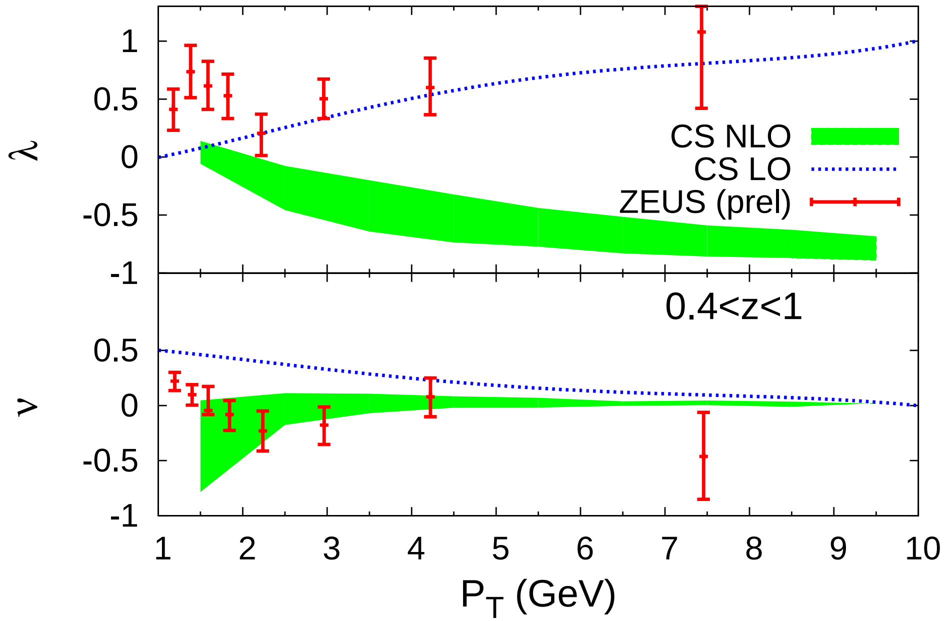



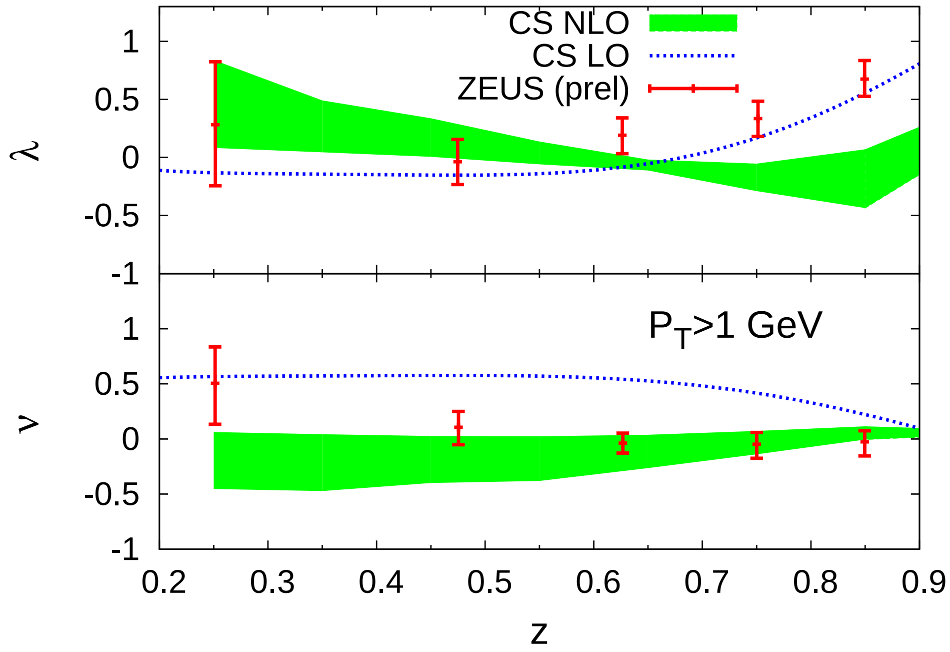

- LO color-singlet plus color-octet calculation by Beneke, Krämer, Vanttinen (1997) using Beneke-Krämer (1996) matrix elements from fits to the Tevatron data.
- The data for $\lambda$ at high $p_T$ slightly favor the color-singlet prediction.
- The data for $\nu$ at high $p_T$ slightly favor the color-singlet+color-octet prediction.

# NLO Calculations

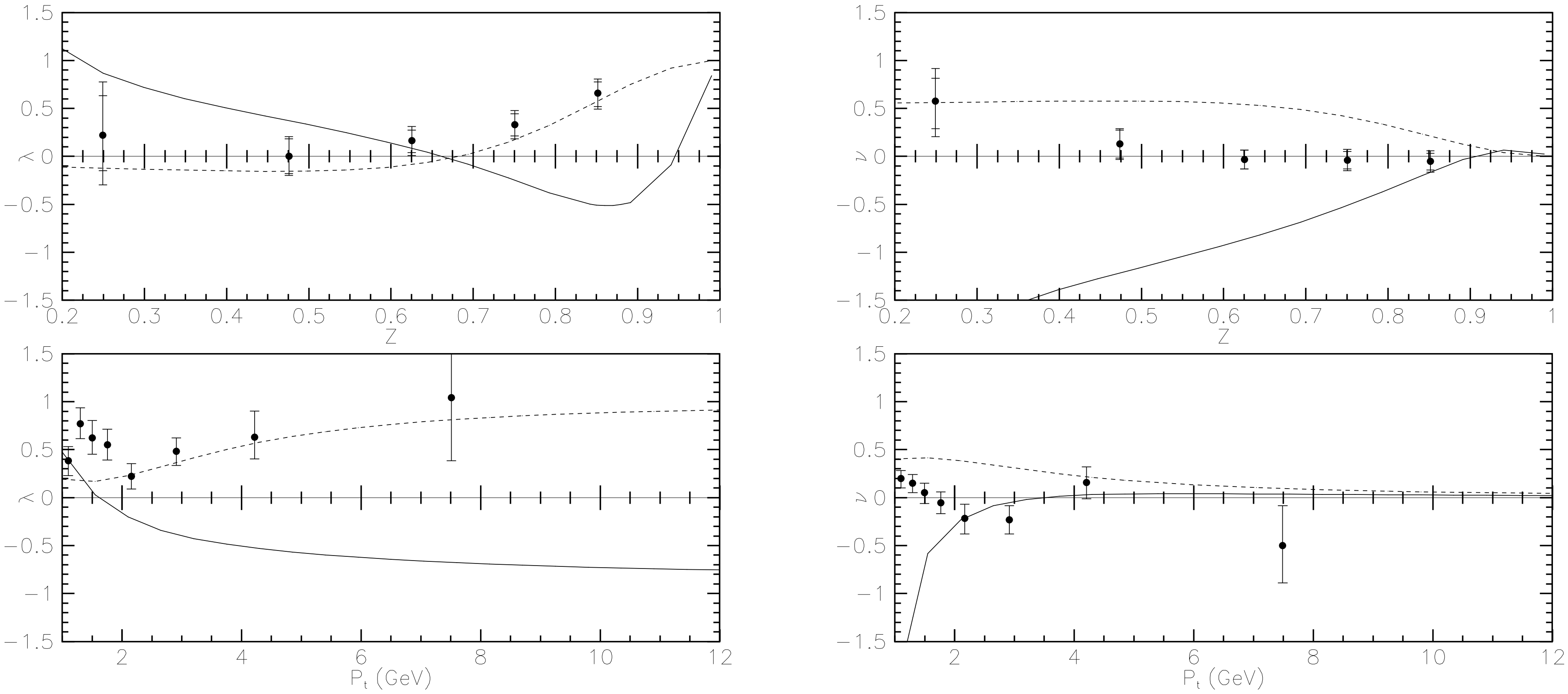
- NLO calculations have a significant effect on the color-singlet polarization predictions.
  - LO calculation: Beneke, Krämer, Vanttinen (1997).
  - NLO calculations: Artoisenet, Campbell, Maltoni, Tramontano (2009); Chang, Li, Wang (2009).

Artoisenet, Campbell, Maltoni, Tramontano:



Chang, Li, Wang:



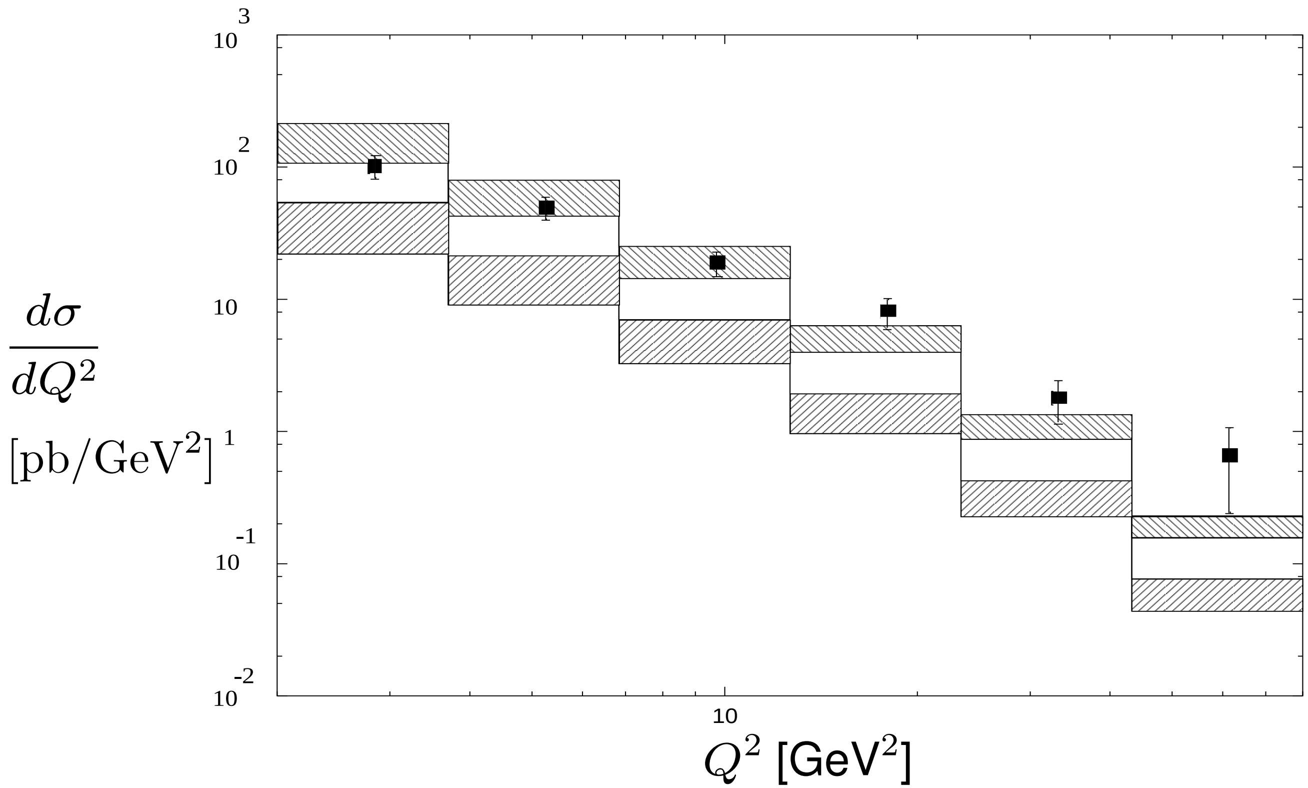

Dotted lines are LO. Solid lines are NLO.

- The NLO calculations agree, except that Chang, Li, and Wang find a more negative result for $\nu$ vs. $z$ than Artoisenet, Campbell, Maltoni, Tramontano.
- The NLO color-singlet contribution alone cannot explain the data for $\lambda$ at large $p_T$ or large $z$.
- Would a color-octet contribution bring theory into agreement with data?

# $J/\psi$ Production in DIS at HERA

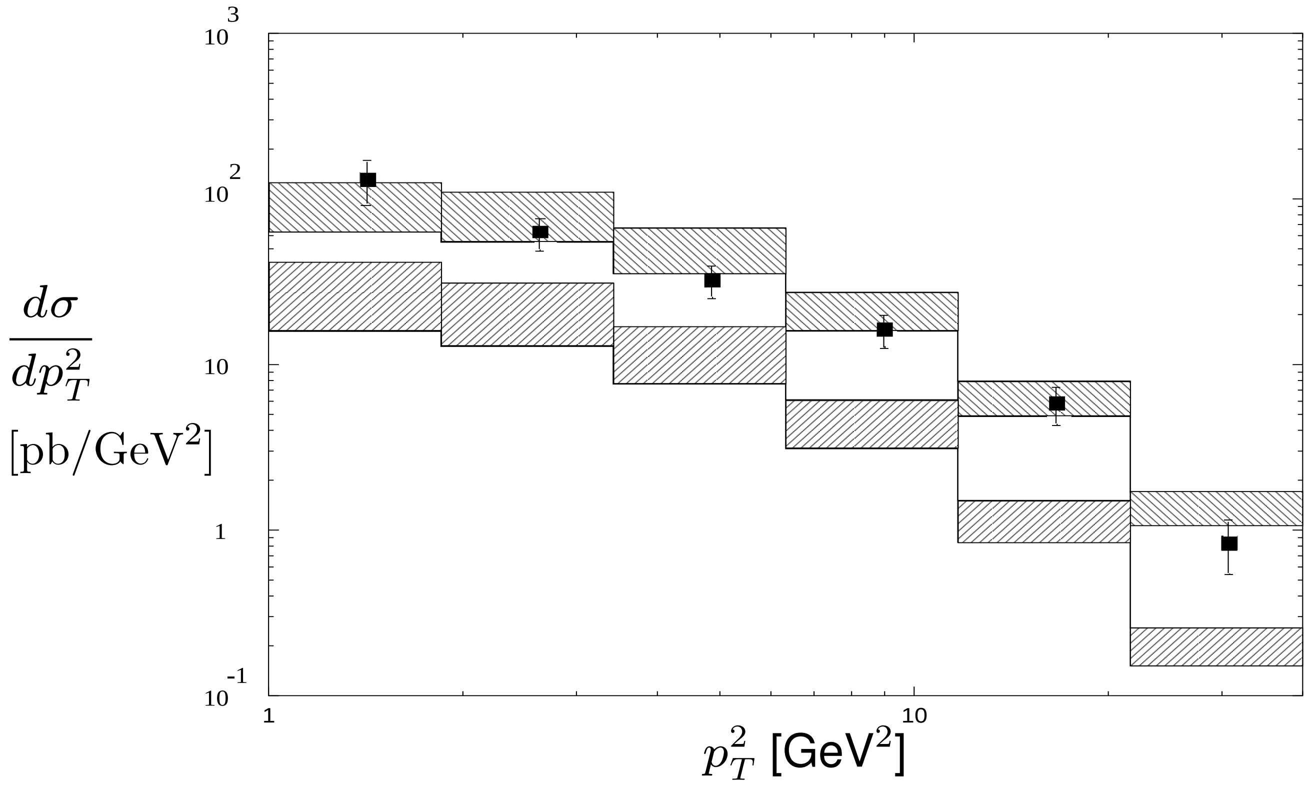
- Note that NLO calculations are not yet available for this process.
- The NRQCD prediction (Kniehl, Zwirner (2001)) uses matrix elements extracted from the Tevatron data (Braaten-Kniehl-Lee (1999)).

- The H1 (1998) data plotted as a function of $Q^2$ favor the NRQCD prediction over the color-singlet-model prediction. ($Q$ is the virtual-photon momentum.)



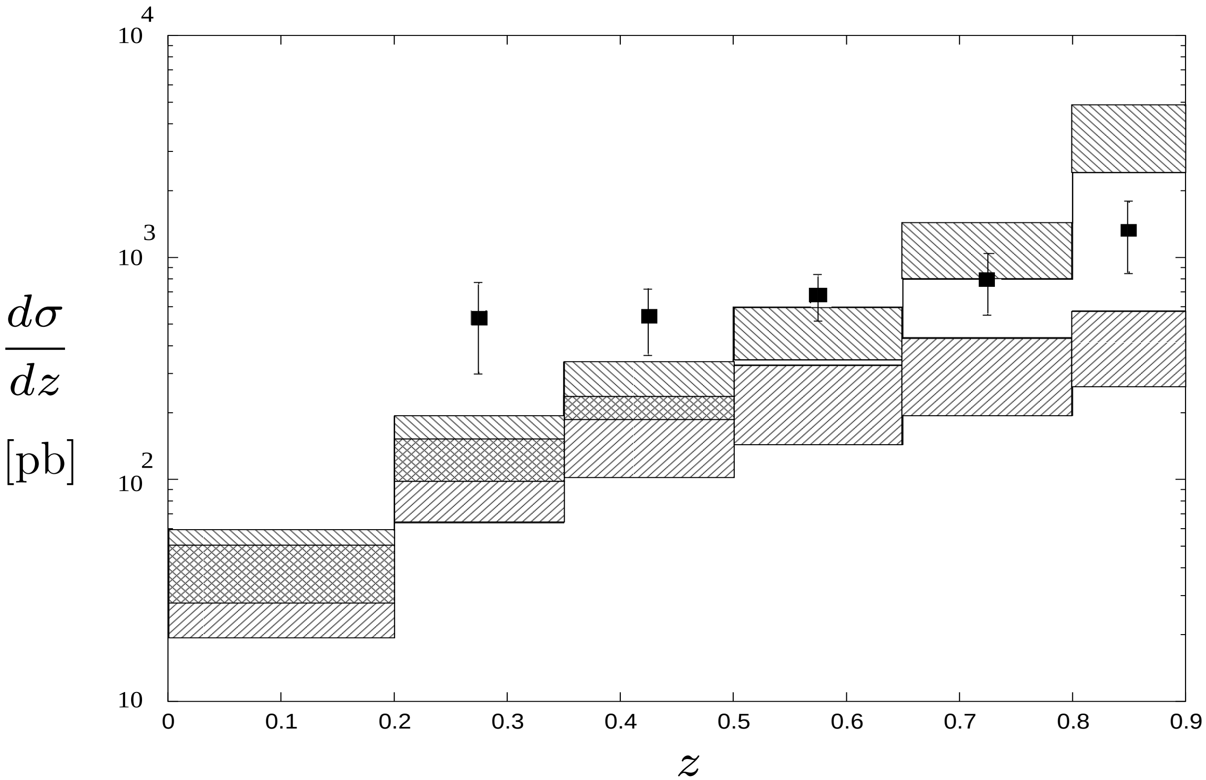

H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The H1 (1998) data plotted as a function of $P_T^2$ favor the NRQCD prediction over the color-singlet-model prediction. $p_T$ is the transverse momentum of the $J/\psi$.




H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

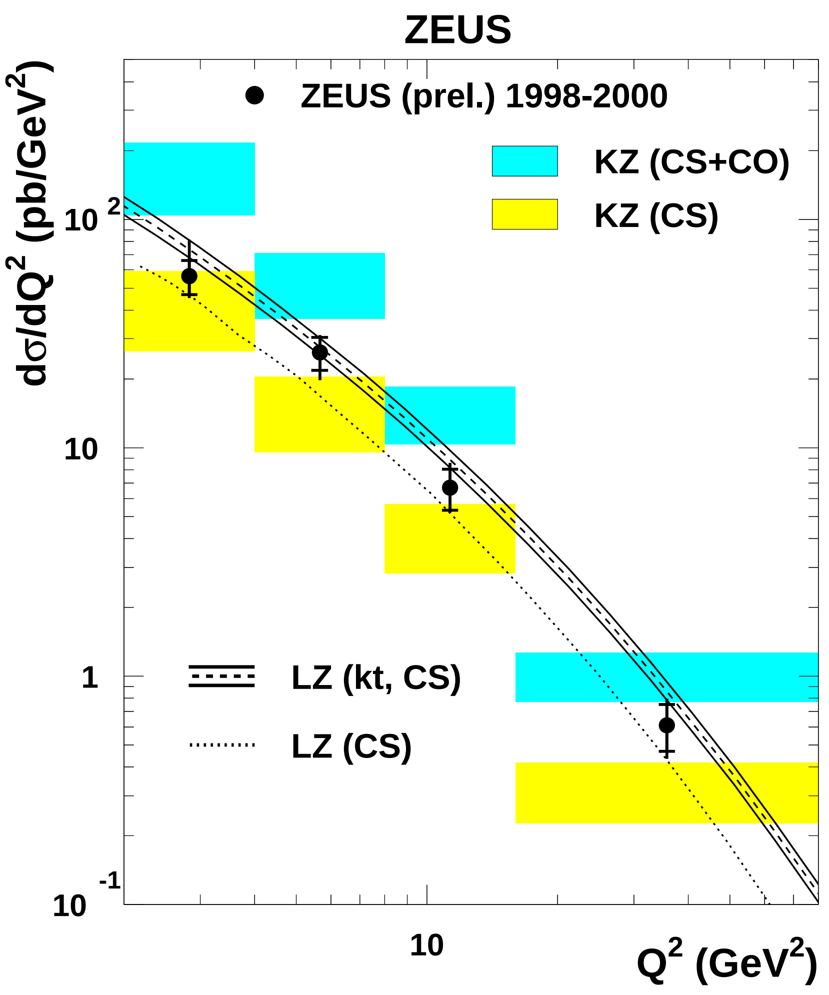
- The H1 (1998) data plotted as a function of $z$ do not agree well with either the NRQCD prediction or the color-singlet-model prediction. ($z$ is the energy fraction of the $J/\psi$.)
- The data do not show the expected color-octet rise at $z = 1$.
  Resummations of the $\alpha_s$ and $v$ expansions are needed.

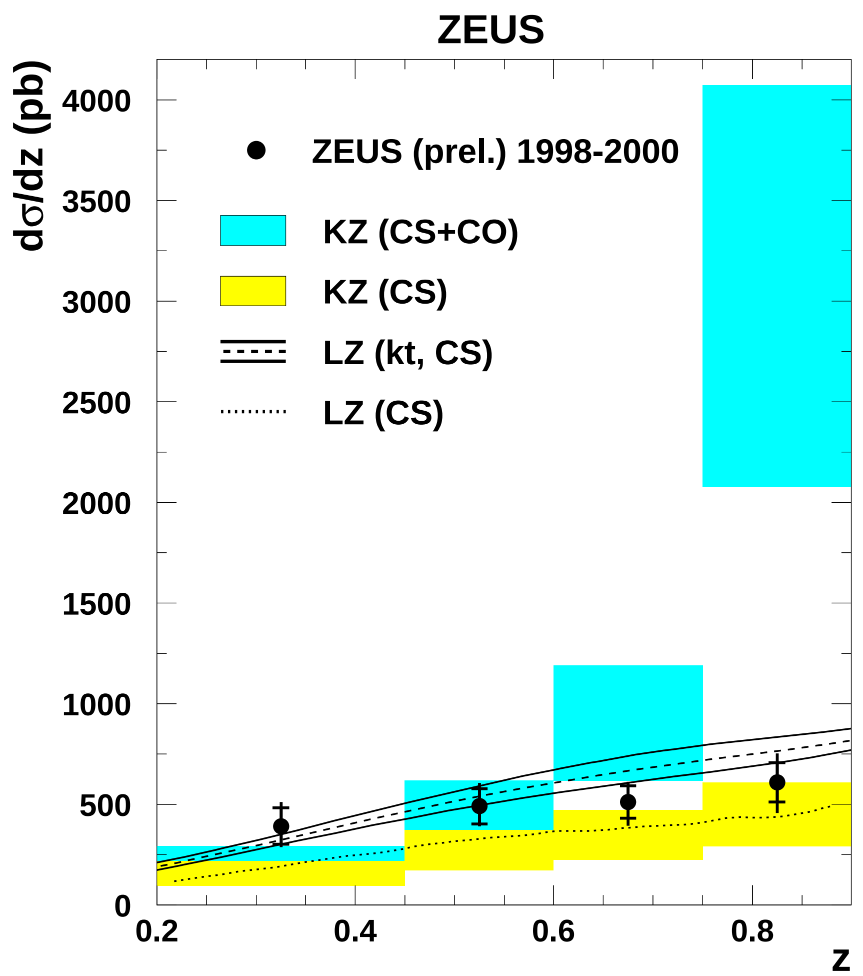



H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).

- The ZEUS data are systematically lower than the H1 (1998) data and agree less well with the NRQCD prediction (but have larger error bars).

- The data plotted as a function of $z$ do not show the expected color-octet rise at $z = 1$.




$k_T$-factorization curves from A.V. Lipatov and N.P. Zotov (2003).

# Factorization in Exclusive Quarkonium Production

- GTB, Garcia i Tormo, Lee (2008): Proof of factorization for the simpler case of exclusive quarkonium production.
  - Avoids the difficulties of additional gluons or heavy quarks in the phase space near the quarkonium.
  - Factorization holds in $B$-meson decays to a quarkonium plus a light meson up to corrections of order $m_c v/m_b$.
  - Factorization holds in $e^+e^-$ annihilation to two charmonia up to corrections of order $m_c^2 v^2/s$.
  - The essential idea:

    In the production CM frame, the four-momenta of the quarkonium constituents are approximately proportional (corrections of order $m_c v/\sqrt{s}$).
  - Soft gluons decouple from a color-singlet object whose constituents have four-momenta that are proportional.
- These proofs are valid only for processes in which there is no helicity flip.
  - For helicity-flip processes, such as $e^+e^- \to J/\psi + \eta_c$, NRQCD factorization is still believed to hold.
  - But the short-distance coefficient contains two scales: $Q$ and $m$.

# Exclusive Double-Charmonium Production at Belle and BaBar

## Comparison of LO Calculations with Experiment

- Experiment
  Belle (2004): $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  BaBar (2005): $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

- NRQCD at LO in $\alpha_s$ and $v$
  Liu, He, Chao (2002): $\sigma[e^+e^- \to J/\psi + \eta_c] = 5.5$ fb.
  Braaten, Lee (2003): $\sigma[e^+e^- \to J/\psi + \eta_c] = 3.78 \pm 1.26$ fb.
  Hagiwara, Kou, Qiao (2003): $\sigma[e^+e^- \to J/\psi + \eta_c] = 2.3$ fb.
  The three calculations employ different choices of $m_c$, NRQCD matrix elements, and $\alpha_s$.
  Braaten and Lee include QED effects.
  Confirmed by Brodsky, Ji, and Lee in light-front QCD for zero relative motion between the heavy $Q$ and $\bar{Q}$.

- Exclusive process: no color-octet contributions.

- The color-singlet matrix elements at LO in $v$ are determined from $\eta_c \to \gamma\gamma$ and $J/\psi \to e^+e^-$.

- There are also disagreements between LO NRQCD predictions and experiment for $J/\psi + \chi_{c0}$ and $J/\psi + \eta_c(2S)$:

| | $J/\psi + \eta_c$ | $J/\psi + \chi_{c0}$ | $J/\psi + \eta_c(2S)$ |
|---|---|---|---|
| $\sigma \times B_{>2}$ (fb) [Belle (2004)] | $25.6 \pm 2.8 \pm 3.4$ | $6.4 \pm 1.7 \pm 1.0$ | $16.5 \pm 3.0 \pm 2.4$ |
| $\sigma \times B_{>2}$ (fb) [BaBar (2005)] | $17.6 \pm 2.8^{+1.5}_{-2.1}$ | $10.3 \pm 2.5^{+1.4}_{-1.8}$ | $16.4 \pm 3.7^{+2.4}_{-3.0}$ |
| $\sigma$ (fb) [Liu, He, Chao (2002)] | 5.5 | 6.9 | 3.7 |
| $\sigma$ (fb) [Braaten, Lee (2003)] | $3.78 \pm 1.26$ | $2.40 \pm 1.02$ | $1.57 \pm 0.52$ |
| $\sigma$ (fb) [Hagiwara, Kou, Qiao (2003)] | $2.3$ | | |

## Corrections of NLO in $\alpha_s$ to $e^+e^- \to J/\psi + \eta_c$

- An important step in resolving the discrepancy:
  Zhang, Gao, Chao (2005) found that corrections at NLO in $\alpha_s$ yield a $K$ factor of about 1.96.
- Confirmed by Gong and Wang (2007).
- Not enough by itself to bring theory into agreement with experiment.
- Zhang, Ma, Chao (2008): In the cases of $\sigma[e^+e^- \to J/\psi(\psi(2S)) + \chi_{c0}]$, large $K$ factors ($\sim 2.8$) may bring theory into agreement with experiment.

## Relativistic Corrections to $e^+e^- \to J/\psi + \eta_c$

- Relativistic corrections $\sigma[e^+e^- \to J/\psi + \eta_c]$ can come from two sources:
  - Direct corrections to the process $e^+e^- \to J/\psi + \eta_c$ itself,
  - Indirect corrections that enter through the matrix element of leading order in $v$.
    Appear when $\Gamma[J/\psi \to e^+e^-]$ is used to determine the matrix element because of relativistic corrections to the theoretical expression for $\Gamma[J/\psi \to e^+e^-]$.
- Relativistic corrections depend on matrix elements of higher order in $v$.
- GTB, Kang, Lee (2006): Determine matrix elements of higher order in $v$ by making use of a potential model.
  - Not really a model because the use of potentials follows from a rigorous expansion in powers of $v$ (pNRQCD).
    The static $Q\bar{Q}$ potential is known from lattice simulations.
  - First determination of these matrix elements with small enough uncertainties to be useful.
- GTB, Chung, Kang, Kim, Lee, Yu (2006): Corrections at NLO in $\alpha_s$ plus relativistic corrections may bring theory into agreement with experiment.
- Confirmed by He, Fan, Chao (2007).
- GTB, Chung, Kang, Lee (2007): New determination of the matrix elements of LO and NLO in $v$ with a detailed error analysis.

- New Calculation of $\sigma[e^+e^- \to J/\psi + \eta_c]$
  (GTB, Chung, Kang, Lee, Yu (2007))
  - Makes use of the improved matrix elements from GTB, Chung, Kang, Lee (2007).
  - Resums relativistic corrections that arise from the potential-model quarkonium $Q\bar{Q}$ wave function.
    * Uses the surprising relation $\langle v^{2n}\rangle = \langle v^2\rangle^n$.
  - Uses the results of Zhang, Gao, and Chao (2005) for the corrections of NLO in $\alpha_s$.
  - Includes the interference between the relativistic corrections and the corrections of NLO in $\alpha_s$.
  - Includes a detailed error analysis

$$\sigma_{\rm tot} = 17.6^{+0.8+5.3+0.7+3.9+0.7+2.8+1.6+1.4+1.9+1.32+1.89}_{-0.9-3.7-0.7-3.0-0.7-2.9-1.5-1.1-2.0-1.32-1.89}\ {\rm fb} = 17.6^{+8.1}_{-6.7}\ {\rm fb}$$

- The corrections to the NRQCD factorization formula are nominally quite small: $\sim (m_c v^2)^2/(s/4) \approx 7\%$.

- $\sigma_{\rm tot}$ consists of

| | |
|---|---|
| $5.4$ fb | Leading order in $\alpha_s$ and $v$ (including indir. rel. corr., but without QED contribution) |
| $1.0$ fb | QED contribution |
| $2.9$ fb | Direct relativistic corrections |
| $6.9$ fb | Corrections of NLO in $\alpha_s$ |
| $1.4$ fb | Interference between rel. corr. and corr. of NLO in $\alpha_s$ |
| $17.6$ fb | Total |

- The individual relativistic corrections are all of order $v^2 \approx 0.3$, but they all go in the same direction.
  - Indirect relativistic corrections are about $37\%$ per quarkonium.
  - Direct relativistic corrections are about $40\%$.
  - The corrections of higher order in $v$ from the resummation are about $13\%$.
  - The $v$ expansion seems to be under control.
- Result for $\sigma[e^+e^- \to J/\psi + \eta_c]$ confirmed, within uncertainties, by He, Fan, Chao (2007), using a fit to decay data to determine the LO and NLO NRQCD matrix elements.

- Theory and experiment agree within uncertainties:
  - Theory: $\sigma[e^+e^- \to J/\psi + \eta_c] = 17.6^{+8.1}_{-6.7}\,\mathrm{fb}$
  - Belle: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  - BaBar: $\sigma[e^+e^- \to J/\psi + \eta_c] \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.
- *Caveat:* $B_{>2}$ is not known.
  - Could be as small as $0.5$–$0.6$.
  - Even so, the error bars of theory and the BaBar experiment overlap.

## Light-Cone Calculations

Bondar and Chernyak (2005)
Braguta, Likhoded, Luchinsky (2006)
Braguta (2007)

- In principle, calculations in the light-cone formalism are a rigorous approach to exclusive double-charmonium production in $e^+e^-$ annihilation.
- LO light-cone calculations automatically sum a class of relativistic corrections and include some corrections that would be NLO in $\alpha_s$ in the NRQCD approach.
- Agreement with experimental results has been achieved using the light-cone approach.
- Results depend strongly on modeling of the charmonium light-cone distributions.
  - Low moments of the light-cone distributions can be constrained by sum rules [Braguta (2007)].
- Modeling is not necessary.
  - Relativistic corrections can be resummed without modeling in the NRQCD by making use of potentials from lattice simulations.
  - Contributions from high-momentum tails of the light-cone distributions need not be modeled because they are calculable in perturbation theory in NRQCD [GTB, Kang, Lee (2006)].

# Inclusive Double $c\bar{c}$ Production at Belle

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X)$$

- Belle (2002):

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) = 0.59^{+0.15}_{-0.13} \pm 0.12.$$

- LO color-singlet model
  (Cho, Leibovich (1996); Baek, Ko, Lee, Song (1997); Yuan, Qiao, Chao (1997)):

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) \approx 0.1.$$

- There is a significant disagreement between experiment and the LO color-singlet model.

## $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$

- Experiment and LO theory disagree.
  - Belle (2009): $\sigma(e^+e^- \to J/\psi + c\bar{c} + X) = 0.74 \pm 0.08^{+0.09}_{-0.08}$ pb.
  - LO Theory (color-singlet): $\sigma(e^+e^- \to J/\psi + c\bar{c} + X) =$ 0.10–0.27 pb.
    Large renormalization-scale dependence.
- Liu, He, Chao (2003): Two-photon contributions are only about $23$ fb.
- Color-octet contributions to $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$
  (Liu, He, Chao (2004)):
  - About $11$ fb total.
  - From $\sigma(e^+e^- \to J/\psi + c\bar{c})$ $(7\%)$ and $\sigma(e^+e^- \to \chi_{c1} + c\bar{c})$ $(32\%)$.
- He, Fan, Chao (2007): Direct relativistic corrections to $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$ are only about $31\%$.

- NLO calculation of the color-singlet contribution to $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$ (Zhang and Chao (2007)):
  - Find a $K$ factor of about $1.8$.
  - Taking into account QED corrections, two-photon processes, feeddown from $\psi(2S)$ (the largest effect) and $\chi_{cJ}$, and LO color-octet corrections, they obtain

    $$\sigma(e^+e^- \to J/\psi + c\bar{c} + X) = 0.53^{+0.59}_{-0.23}\ \text{pb} \qquad (\mu = \sqrt{s}/2).$$

    The uncertainties come from $m_c$.
  - Calculation confirmed by Gong and Wang (2009).
  - Resolves the discrepancy between theory and experiment, but theoretical uncertainties are large.

- Nayak, Qiu, and Sterman (2007, 2008): there could be a nonperturbative enhancement to production of $J/\psi + c\bar{c}$ when the $c$ or the $\bar{c}$ is co-moving with the $J/\psi$.
  - This effect can't be calculated reliably in perturbation theory.
  - Its size must be determined experimentally.

## $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})$

- Experiment and the LO color-singlet contribution are in rough agreement.
  - Belle (2009):
    $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.43 \pm 0.09 \pm 0.09$ pb.
  - LO color-singlet contribution (Ma, Zhang, Chao (2008); Wang (2003)):
    $\sigma(e^+e^- \to J/\psi + gg) = 0.36$ pb ($\mu = \sqrt{s}/2$)
- LO color-octet contribution
  (Braaten and Chen (1996); Cho and Leibovich (1996); Yuan, Qiao, and Chao (1997); Chao and Hao (2003); Baek, Ko, Lee, and Song (1998); Wang (2003)):
  $\sigma(e^+e^- \to J/\psi + g) = 0.28$ pb.
  Based on LO Tevatron extractions of the ${}^1S_0$ and ${}^3P_J$ color-octet NRQCD matrix elements.
- NLO color-singlet contribution
  (Ma, Zhang, and Chao (2008); Gong and Wang (2009)):
  $\sigma(e^+e^- \to J/\psi + gg) = 0.53^{+0.12}_{-0.09}$ pb ($\mu = \sqrt{s}/2$).
  - Includes the effects of feeddown from the $\psi(2S)$: $0.14$ pb.
  - The uncertainty is from $m_c$ only. Probably an underestimate.
- Relativistic corrections to the color-singlet contribution to $\sigma(e^+e^- \to J/\psi + gg)$
  (He, Fan, Chao (2009)): $0.15 \pm 0.05$ pb.
  Computed using matrix elements from Bodwin, Chung, Kang, Lee and Yu (2008).

- NLO color-octet contribution (Zhang, Ma, Wang, and Chao (2009):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) \approx 1.5 \text{ pb.}$$

  - Uses Krämer (2001) matrix elements from LO fits to Tevatron data.
  - Apparent discrepancy with data, but use of Butenschön and Kniehl (2011) NLO matrix elements and use of the scale $\mu = \sqrt{s}/2$ instead of $\mu = m_c$ gives

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.33 \pm 0.10 \text{ pb.}$$

## Color-singlet plus color-octet contributions to $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})$

- Sum of known contributions, including the color-singlet-process feeddown from the $\psi(2S)$and relativistic correction:

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 1.01^{+0.16}_{-0.14} \text{ pb.}$$

  - About a $2\sigma$ discrepancy with the Belle (2009) result.
  - Theoretical uncertainties are probably underestimated.

New calculation of the sum of color-singlet and color-octet contributions (Butenschön and Kniehl (2011)):

$$\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c})) = 0.70^{+0.35}_{-0.17}\ \text{pb}.$$

- NRQCD matrix elements determined in a global fit to KEKB, LEP II, RHIC, HERA, Tevatron, and LHC data.
- This calculation and the Belle (2009) data agree within error bars.
- However, the calculation does not include the color-singlet-process feeddown from the $\psi(2S)$ and relativistic corrections.

  They would increase this result by about $0.29$ pb, leading to a $2\sigma$ discrepancy with the Belle (2009) result.

## Comments

- Note that BaBar (2001) obtained

$$\sigma(e^+e^- \to J/\psi + X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb},$$

in contrast with the value that can be inferred from the latest Belle (2009) measurements

$$\sigma(e^+e^- \to J/\psi+X) = \sigma(e^+e^- \to J/\psi+c\bar{c}+X)+\sigma(e^+e^- \to J/\psi+X(\text{non-}c\bar{c})) = 1.17\pm0.12^{+0.13}_{-0.12}$$

- It is important for BaBar to check the Belle results for $\sigma(e^+e^- \to J/\psi + c\bar{c})$ and $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c}))$.

## Effect of NLO calculations on the ratio

- The NLO calculations agree with the Belle (2002) result, within error bars:

$$\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) \approx 0.35^{+0.23}_{-0.17}\,.$$

  - This computation uses
    $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$: Ma, Zhang, and Chao (2008) result
    $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c}))$: Butenschön and Kniehl (2011) result
    plus color-singlet-process feeddown from $\psi(2S)$ and relativistic corrections.
  - It would be good to have a detailed error analysis for the theoretical prediction.

## Summary

- The NRQCD factorization approach provides a systematic method for calculating quarkonium production (and decay) rates as double expansions in powers of $\alpha_s$ and $v$.
- NRQCD factorization for inclusive production rates has not yet been established.
- There is now a proof of NRQCD factorization for exclusive charmonium production in $B$-meson decays and $e^+e^-$ annihilation.
- NRQCD factorization has enjoyed a number of successes:
  - quarkonium production at the Tevatron,
  - $J/\psi$ production at RHIC,
  - $J/\psi$ production at the LHC,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - inelastic $J/\psi$ photoproduction at HERA,
  - $J/\psi$ production in DIS at HERA,
  - exclusive double-charmonium production at Belle and BaBar.

- Theory and experiment are no longer in obvious conflict for $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)$. Theoretical uncertainties are large.
- Theory and experiment for $\sigma(e^+e^- \to J/\psi + X(\text{non-}c\bar{c}))$ disagree at the $2\sigma$ level.
- The situation is still ambiguous for polarization in inelastic $J/\psi$ photoproduction at HERA.
- The disagreement between theory and experiment for quarkonium polarization at the Tevatron presents a serious challenge.
- In a number of cases, corrections of higher order in $\alpha_s$ and $v$ and resummations near kinematic endpoints have proven to be essential to obtain reliable theoretical predictions.
- In many cases, the perturbation expansion converges poorly, and theoretical uncertainties are large.
- The fragmentation approach of Kang, Qiu, and Sterman may help to bring theoretical uncertainties under control.
- Measurements of direct-production cross sections and polarizations would be of great help in understanding production mechanisms.
- We need to make additional measurements beyond $d\sigma/dp_T$ at hadron-hadron colliders in order to pin down the quarkonium production mechanisms.